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GEORGE ALEXANDER OSBORNE, Esq., IN THE CHAIR.

ON THE SENSITIVENESS OF THE EAR TO PITCH AND CHANGE OF PITCH IN MUSIC.

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THE principal aim of the present paper is to bring before the Musical Association some results obtained last autumn from numerous and very careful experiments by Dr. W. Preyer, Professor of Physiology in the University of Jena, because those results are not accessible in an English form, and seem to have a very practical bearing upon tuning and singing.* My subordinate purpose is to exhibit a method of representing pitch and intervals, which shall be more consonant to the habits of musicians than that adopted by Dr. Preyer for the statement of his results. For the idea of this method I am indebted to Mr. Bosanquet, though I have worked it out somewhat differently from him.

Considered arithmetically (as distinguished from physiologically), the pitch of a note depends upon the number of (double, or complete, backwards and forwards) vibrations or *swing-swangs* as De Morgan termed them, made by a particle of air in one second. As this vibration depends on the length of a path described in a given time, we may readily talk of any fraction, or even any incommensurable part of a vibration which would be represented by that fraction, or part, of the path described. A pitch is higher or lower according as this number of vibrations is greater or less.

Considered arithmetically, likewise (again, as distinguished from physiologically), an interval is measured by the ratio of the pitch of the lower to the pitch of the higher note, and is usually represented by an improper fraction in which the larger number is the numerator. The principal large intervals which will be

* *Ueber die Grenzen der Tonwahrnehmung* (on the Limits of the Perception of Musical Tone), Jena, 1876; preface dated 'Autumn 1875,' 8vo. pp. viii. 72.

here considered are the Octave $\frac{2}{1}$, the Fifth $\frac{3}{2}$, the Fourth $\frac{4}{3}$, the major Sixth $\frac{5}{3}$, the major Third $\frac{5}{4}$, the minor Third $\frac{6}{5}$, the minor Sixth $\frac{8}{5}$ and the major Tone $\frac{9}{8}$. The minor Tone $\frac{10}{9}$, the diatonic Semitone $\frac{16}{15}$, and intervals greater than an Octave (for which the fraction exceeds 2), and the intervals involving 7 will not be considered, because Dr. Preyer has not investigated them. But a large number of intervals differing from the above by very small intervals which have no regular names, will require attention.

By means of beats we are probably able, under favourable circumstances, to determine Pitch to the tenth or the hundredth, or even, as Scheibler imagined, to the six-hundredth part of a vibration: that is, under favourable conditions we can appreciate one beat in ten or a hundred seconds of time—or, to take Scheibler's estimate, one beat in ten minutes; and consequently, the interval between two notes with the same exactness, since one beat represents a difference of one vibration in that time. But then subsidiary notes are required, and we must be able to sustain two notes together without any alteration in either for ten or one hundred seconds, and to count the beats accurately for that time.* Under such favourable circumstances, therefore, we can approximately realise the meaning of arithmetical pitch and arithmetical interval. But the perception of beats requires nothing more than the power of hearing. A so-called 'musical ear' or sensitiveness of ear with regard to pitch and change of pitch is not required. The determination may therefore be called mechanical. It is by such mechanical observations alone that we can check the judgment of the ear, and hence arrive at a mode of measuring sensitiveness in estimating the intervals between two notes sounded in succession, and not at the same time. The existence of beats, I may remark, indicates only that the beating tones are not of the same pitch, and does not point out which is the sharper. To determine this we have to make one note sharper or flatter, and then see whether the number of beats increases or diminishes. As they increase by enlarging, and diminish by contracting the interval, this gives a mechanical means of determining which is the sharper. In the case of tuning forks, placing one under the arm for a minute will generally flatten it sufficiently, when two forks are nearly of the same pitch, beating not more than twice in a second, to determine whether the beats increase or slacken in speed: if the former, the warmed fork was already the flatter; if the latter, it was the sharper. Sensitiveness of ear is thus required solely for judging of melodic intervals: that is, of the difference of the pitch of two tones sounded in succession. Before examining this, however, it

* In verifying Appunn's Tonometer in the Loan Exhibition of Scientific Instruments, I counted the beats during 20 seconds. An error of one beat could therefore have only produced an error of one-twentieth of a vibration in a second. Any decent tuning-fork will give audible beats with a neighbouring tone for 5 or 10 seconds, and hence allow of measurements to one-fifth or one-tenth of a vibration.

is necessary to have a means of noting small deviations of pitch in a way which, while it is consonant with the existing habits of musicians, shall rest upon a strictly arithmetical basis.

All musicians are familiar with the Octave, and accustomed to divide the whole range of musical sounds into Octaves. This amounts to selecting a series of tones on the principle of continually multiplying the corresponding number of vibrations by 2. Arithmetically, we are therefore bound to begin with less than 2 vibrations in a second, which, not being multiplied by 2 at all, may be said to commence the *Zero Octave*, and the simplest such number that can be selected is 1 itself.* Then from 2 up to 4, we multiply the former number of vibrations *once* by 2, and have the *first Octave*. At 4, or twice 2, we multiply *twice* by 2, and have the *second Octave*. The Octaves will there correspond with the numbers of vibrations with which they begin, thus—

Oct	0	1	2	3	4	5	6	7	8	9	10	11	12
Vib	1	2	4	8	16	32	64	128	256	512	1024	2048	4096.

And so on.† Supposing the notes C, D, E, &c. to occur in any Octave, the number of the Octave is prefixed: thus 8E means E in the 8th Octave. The Octave being always supposed to begin with C, 8C will be a note having 256 vibrations. The difficulties arising from habits of using different standards of pitch and temperaments are met by assuming some fraction (as $1\frac{1}{32}$) in place of 1 for the initial number of vibrations. But what is commonly called the 'theoretical,' or as I prefer to term it the 'arithmetical' pitch 8C=256 vib is the only one suggested by pure arithmetic. When any other initial pitch is used, we may accent the letters: thus, 8C′=264 vib gives 0C′=33÷32 vib. In Table III. I give the values of 0C′ for such values as 8A′ and 9C′, as cover the whole range of usual vibration of pitch.

This simple contrivance for marking the Octave variation of standard pitch obviates a vast number of difficulties. Observe the following synonyms:—

4C=16 vib means C in the Sub-Contra Octave, or double under-accented or under-lined Great Octave, C_{II} or $\underline{\underline{C}}$ or twice indexed Great Octave C_2, or twice negatively indexed Great Octave $\underline{C}_2$ or C^{-2}, or 32-foot Octave, or triply repeated Great Octave CCC.

5C=32 vib means C in the Contra Octave, or once under-accented or under-lined Great Octave C_I or $\underline{C}$, or once indexed, or once negatively indexed Great Octave C_1, $\underline{C}$ or C^{-1}, or 16-foot Octave, or doubly repeated Great Octave CC.

* This selection of 1 as the initial pitch is made on purely arithmetical grounds. The mode of treating other cases will be explained immediately. There is here no intention of deciding on the best standard pitch for practical purposes.

† See also the bottom of Table I. In higher arithmetic the number of the Octave is the index of the power of 2; that gives the number of vibrations with which the Octave begins. Thus $256=2^8$ begins the 8th Octave.

6C=64 vib means C in the Great or 8-foot Octave, with one large C, without accents or indexes, the lowest note of the Violoncello.

7C=128 vib means C in the Small or 4-foot Octave, with one small *c*, without accents or indexes, the lowest note of the Tenor or Viola da Braccia.

8C=256 vib means C in the once accented or 2-foot Octave, with a small *c* and one acute accent as *c*′, once over-lined *c* as $\bar{c}$, or two small *c*'s as *cc*; commonly called 'middle C,' being on the one leger line between the bass and treble staves.

9C=512 vib means C in the twice accented or twice over-lined or 1-foot Octave, with a small *c* and two acute accents as *c*″, or two lines as $\bar{\bar{c}}$, or three small *c*'s as *ccc*.

10C=1024 vib means C in the thrice accented or $\frac{1}{2}$-foot Octave, with a small *c* and three acute accents or over lines as *c*‴, or $\bar{\bar{\bar{c}}}$.

And so on for higher Octaves, the number of the Octave being 7 more than the number of acute accents or over-lines.

The next division, with which the pianoforte in recent times has rendered all musicians perfectly familiar, is that of the large interval of an Octave into twelve smaller intervals called Equal Semitones, which I shall contract into the word *sem*. Anyone who plays up the chromatic scale on a piano has a sensation of sameness of interval in passing from any one note to the next in succession. But the actual sensation of the interval sem is very indistinct, and few, I apprehend, if any, would venture to tune a piano by that interval only. It happens to be one of those intervals which cannot be exactly expressed by figures, but, far nearer than any human ear can detect, we may speak of it as $\frac{1782}{1682}$ or $1\frac{100}{1682}$,* so that if we know the number of vibrations, or vib of any note, say 8C=256, we have only to multiply it by 100, giving 25600, divide this by 1682 to two places of decimals, giving 15·22, and add it to the vib of the lower or 256, giving 271·22 vib for 8C♯, which is correct to two places of decimals. Hence we obtain a perfectly clear arithmetical conception of a sem or the twelfth part of an Octave.

Now take an imaginary fingerboard of a pianoforte, 40 feet in length, and at the beginning and at the end of each 40 inches introduce a single string which on being struck vibrates exactly 100 in 1682 vib. more than its predecessor. We shall then have a fingerboard which may be represented by the annexed diagram (Table I.),† in which, for convenience, the long line of 40 feet is

* Of course this has been obtained by higher arithmetic, and is a close approximation to the 12th root of 2. Observe that $\frac{1782}{1682}$ gives the well-known approximations $\frac{17}{16}$ (too large) and $\frac{18}{17}$ (too small). It will be seen by Table II., col. B, below, that the vibrations of A are to those of A♯ as 1,681,793 to 1,781,797, which gives the above ratio. A whole Tone is very nearly indeed 10000÷8909, which is obviously less than the Major Tone 9÷8, and greater than the Minor Tone 10÷9.

† When the paper was read, a diagram, having the dimensions here

divided into 12 lines of 40 inches each, and the note which ends any one also begins the next, as indicated by the usual name which is placed at the beginning and end of each line.

But this is still a very rough division. We can readily imagine 9 other strings introduced between the extremes of each line, at equal distances of 4 inches, and giving notes proceeding by equal intervals, which repeated ten times produces a sem. Such an interval, which may be called a *tithe*,* is represented with great exactness by $\frac{1736}{1726} = 1\frac{10}{1726}$. So that to find a tone which is a tithe higher than $8C = 256$, we multiply by 10, giving 2560, and divide, to three places of decimals, by 1726, giving 1·425, and then add this to 256, giving 257·425 vib. The mode of noting this will be given presently.

We must carry this division further, and suppose that between each pair of strings, which give notes separated by a tithe, we introduce 9 other strings separated from each other by 4 tenths of an inch, and proceeding by equal intervals, which repeated 10 times will produce a tithe. Such an interval, which will be naturally called a *cent*, is represented almost precisely by $\frac{1732}{1731} = 1\frac{1}{1731}$, so that to find an interval which is one cent sharper than $8C = 256$ vib we divide 256 by 1731 to 2 places of decimals, giving ·15, and add to 256, giving 256·15 vib.

For many theoretical calculations we require to go further, and find an interval which is the tenth part of a cent, and may be called a *mil*. The ratio of a mil is $\frac{17311}{17310}$ or $1\frac{1}{17310}$. We even find it necessary to imagine an interval which is the tenth part of a mil, and may be called a *dime*, having the ratio $\frac{173101}{173100}$ or $1\frac{1}{173100}$.† Such intervals are, of course, far beyond all power of ear to appreciate when the notes are struck in succession, and can scarcely be heard by beats except under wonderfully favourable circumstances, yet in accumulating a large number of intervals we should often commit very appreciable errors in calculation, if we did not use the mil, and occasionally the dime. But the cent is the practical limit.

In the diagram the tithes are marked by alternate black and blank parallelograms, the upper boundary forming the line considered. The separation of the colours gives the position of each

indicated, was exhibited. It is by no means large or unwieldy, but for the purposes of printing it, the length of the imaginary key-board has been reduced to 48 inches, divided into 12 parts of 4 inches each, that is, it appears as one-tenth of its actual size. The statement of the actual dimensions is, however, retained in the text.

* *Tithe*, of course, means a *tenth*, but the word *tenth* itself had to be omitted because it is the name of well-known intervals, the major Tenth, or $\frac{5}{2}$, and the minor Tenth, or $\frac{12}{5}$.

† To reduce these relations to one common standard, suppose that the starting note makes 1,000,000 vibrations in some known interval of time. Then a note which is a sem higher than the starting note makes 59,462 vibrations more in the same time; a note which is a tithe higher makes only 5,793 vibrations more; one a cent higher only 578 more; one a mil higher only 58; and one a dime higher only 6 vibrations more than the starting note, in the same time.

tithe as a geometrical line. The cent and mil are determined by a scale, easily drawn, each cent being represented by a space of 4 tenths and each mil by one of 4 hundredths, that is 1 twenty-fifth of an inch; but a dime, occupying $\frac{1}{250}$ inch, is far too small to be noted even on a keyboard of 40 feet to the Octave. To mark the position of any particular string or note terminating a cent or mil, a pointer is used, in the shape of half a square, or two half-squares superposed.* Where the vertical edge of the pointer cuts the line of sem we have a mathematical point which exactly shows the interval, even in a large diagram.

Any pitch can therefore be fixed to the nearest mil or dime by stating that it is so many Octs, so many sems, so many tithes, so many cents, and so many mils, or so many dimes: as 8 Oct, 5 sem, 3 tithes, 7 cents, 9 mils, and 6 dimes. But since 8 Oct contain 8 times 12 or 96 sem, and 96 sem + 5 sem is 101 sem, this result can be more conveniently represented by the decimal fraction 101·3796 sem or to the nearest cent (in which case a cent must be added if there are 5 mils or more over) as 101·38 sem.†

It is evident that, by a laborious calculation, this number of sem would enable us to find the exact number of vib, just as a number of vib would give us the complete result in sem. By Table II. this calculation is reduced to simple multiplication and long division, which, though rather lengthy when we wish to go as far as dimes, is really shorter and simpler than the usual processes as far as cents, and involves no references to other tables.

In writing any note, then, we may use such a form as 101·381 sem, or, since 5 sem corresponds to equally tempered F, and 96 sem to 8 Octaves, we might write 8 F + ·381, that is, 381 mils sharper than 8 F to the standard 8 C = 256 mil. Suppose, however, it was more than half a sem, or 500 mils sharper, such as 101·683 sem or 8 F + ·683, it would be more intelligible generally to consider it as 8 F♯ − ·317, that is, as a flat F♯, to which it is nearer, rather than a sharp F from which it is more distant. But, suppose that we want to represent the pitch with reference to another standard as 8 C′ = 264 vib. Then we must find the interval $\frac{264}{256} = 1\frac{1}{32}$ = ·533 sem; and having laid down that C′ D′ &c. now mean notes sharper than C D &c. by ·533 sem, we have 8 F′ = 8 F + ·533 first, and hence 8 F′ − ·152 = 8 F + ·381, so that the pitch will be expressed as a flat F′ rather than as a sharp F. The difference ·152 sem is clearly found by deducting ·381 from ·533. Similarly 101·683 sem = 8 F♯ − ·317 = 8 F + ·683 = 8 F′ + ·150,‡ giving a sharp F′ instead of a flat F♯. It is quite evident, however, that if we confine our notation to figures, as 101·381 sem, or 101·683 sem, we avoid all confusions depending on variable pitch, and obtain a clear and distinct representation of any pitch, from which we

* In order to be visible, the pointers were really about $\frac{4}{10}$-inch wide at the bottom, but the measurements were determined by the vertical side.

† See examples of this mode of expressing pitch in Tables IV. and V.

‡ In Table III. a means is given for finding 0 C from any value of 8 A or 9 C′ that is usually employed.

can form any number of intervals following any given relations of intonations with ease and certainty. Supposing we stopped at cents, the ideal Octave would consist of 1,201 strings each 4 hundredths (or one-twentyfifth) of an inch separated from its nearest neighbour, and covering 40 feet in length, as in the diagram (reduced to 48 inches in Table I.). If this were continued for 12 octaves, we should have a keyboard 480 feet long, containing 14,401 strings. Now, preposterous as this may appear, it is a very close representation of the actual constitution of the cochlea in the internal ear, which is of course microscopic in detail. This contains (according to the latest calculations of Hensen, as cited by Dr. Preyer, p. 41), about 16,400 capillary nerve fibres, which act as the single strings of our ideal monster pianoforte, and are struck, not by hammers, but by the vibrations of a peculiar fluid in which they float. When we ascend on a piano, we pass from the left to the right by degrees, and we fix the pitch by stopping at a certain digital. Thus also in the ear we pass over the nerve fibrils in succession, and fix the pitch by stopping at one of them. The only difference appears to be this: that on our ideal piano we have made the number of strings in each octave, and the interval between the notes, exactly the same, while in the ear it is probable that the intervals differ, and that the number of nerve fibrils is different in each octave. We must also remember that very few musical tones are simple, so that the motion of the fluid affects a number of fibrils at the same time and with unequal force. The old experiment of raising the dampers on a flat piano, and shouting a vowel on to the sounding board, which will be in a small, but appreciable fraction of a second re-echoed from the strings, exhibits the action of the whole mechanism with sufficient exactness to make it comprehensible.

The arithmetical pitch of a compound note is that of its lowest constituent or partial simple tone, but all the other partials help to give individuality to the sensation, and it is found that differences in the quality of tone, in the loudness, or duration of the notes, or even in the length of silence between two notes, materially affect the estimation of pitch and interval by the ear. It is therefore for many purposes advisable to construct a table which gives the exact interval between the lowest and any other partial. This I have done in Table IV. at the end of this paper, up to 64 reckoned to dimes. By adding these sem to the sem of the note, as far as we please (it is seldom necessary to proceed beyond 12 or 16, or higher than 12 C=144 sem) we obtain a very precise indication of the real phenomenon presented by sounding a compound musical note. Thus, if we sound a strong reed tone of which the lowest partial is 77 sem or 6 F, we really hear to the nearest cent the simple tones 77, 89, 96·02, 101, 104·86, 108·02, 110·69, 113, 115·04, 116·86, 118·51, 120·02, 121·41, 122·69, 123·88, 125: that is, 6F, 7 F, 8 C+·02, 8 F, 8 A−·14, 9 C+·02, 9 D♯ −·31, 9 F, 9 G·04, 9 A−·14, 9 A♯ +51, 10 C+·02, 10 C♯ +41, 10 D♯ −·31, 10 E−·12, 10 F. And it is evident that these numerous fixed points, or at least the first 8 of them, which do not in general beat together, will be very instrumental in determining the sensation of pitch. In fact, for simple or nearly simple tones the sensation of pitch is found to be very imperfect, and for different qualities of tone it is apt to be very different, even when their arithmetical pitch is identical.* On the diagram (Table I.) I have given the odd-numbered partials only, reduced to the same octave, because the evenly numbered partials will always be octaves of some of these. The marks for these are double wedges with the number of the partial placed near them, and, if they correspond to any named note, its name is placed below. A single glance at their position will show that the harmonic theory of the scale implies a selection of tones as well as any other. I have also marked by single wedges (when they are not in the harmonic series) and with names below, a series of 52 just Notes arranged systematically, with the pitches attached, in Table V. These exhibit in a striking manner the differences of a great comma of 235 mils, an ordinary comma of 215 mils, and a skhisma of 20 mils which frequently occur, and hence become the starting points for temperament.

Having spent so much time over the representation of intervals, by a musicianly process, on an arithmetical basis, I use the sem, cents and mils thus obtained to represent the results obtained by Prof. Preyer in comparing the actual physiological powers of exceptionally good and highly cultivated ears, in approximating to these arithmetical results.

First, *What are the lowest musical tones audible?* that is, *What is the smallest number of vibrations in a second which can be heard as one continuous tone?*

For this purpose Dr. Preyer had an instrument constructed by Herr Appunn which was shown in the Loan Collection of Scientific Apparatus in 1876. It consists of a series of metal tongues vibrating from 8 to 32 times in a second, and increasing by 1 vibration for each note, and then increasing by 2 vibrations for each note to 64 vib, and then by 4 at a time to 128 vib.† It must be remembered that the resulting tones are very compound, and give numerous disagreeable beats. But it is the lowest partial only on which attention must be fixed, and Dr. Preyer found that if the wind were allowed to die out gradually this lowest tone was heard best just at the moment of disappearance. The result is, that Dr. Preyer could hear a tone of 15 vib, and could not hear one of 13 vib. For 14 he was uncertain. Most ears could not hear below 24 vib.‡ When I played down the

* See Dr. Stone's observations on the difficulty of distinguishing a difference of pitch in the Oboe and Clarinet, and Mr. Hipkins's remark on pianos tuned to the same pitch but having different qualities of tone. Dr. Stone, in *Concordia*, 'On Tuning an Orchestra.'

† This instrument was purchased by the South Kensington Museum at the close of the exhibition.

‡ Dr Preyer gives the following as the final result of experiments repeated hundreds of times when the listener did not know what number of

series on Appunn's instrument, most persons heard quite well to 32 vib but after that, somewhere about 30 vib (varying with different persons), the tone suddenly seemed to jump up an Octave; this arose from the lowest partial ceasing to be audible. Those who, applying the ear carefully to the box of the instrument, seemed to hear a deep tone, were generally hearing the Octave above that which they really fancied they heard. If the lowest partial were strongly reinforced by resonators, possibly the lower tone would be heard better. But the means of producing easily audible simple tones, which are lower than 23 to 15, seem to be not as yet discovered.*

The result is, that the existence of the *simple* tone 4 C=16 vib is a very doubtful, but it may be musically effective when associated as a compound tone with higher partials, and that 5 C= 32 vib. is the lowest compound tone of which the lowest partial can be made musically effective.

The next question is, *What is the highest musical tone that can be heard?*

vibrations he had to hear, but was somewhat acccustomed to observe, and listened with his ear close to the wood of the instrument as the wind was allowed to die out:—

8, 9. No musical tone; an intermittent frictional noise is heard, and the intermittences can be counted.

10, 11, 12, 13, 14. No musical tone; the tremor is felt, and the vibrations are visible; the rattle is feebler.

15. No musical tone; some perceive an obscure sensation of sound.

16, 17, 18. The sensation of musical tone commences; in addition to the tremor of the air, which is sensible to the touch, many hear an obscure sound.

19, 20. With many the sensation of tone is distinct; the tone itself buzzes gently.

21, 22. Many hear a humming tone.

23, 24. Everyone whose sensation of hearing is not impaired now hears a very deep, mild, beautiful tone.

25, 26, 27, 28, 29, 30. As the pitch of the tone increases it becomes less easily heard, because the length of the time for which it lasts, as the wind dies off, becomes shorter, but it is distinct.

31, 32. The tone is still distinct, but short.

34, 36, 38. The tone is very short and difficult to hear.

40. Impossible to hear anything, because the vibrations of the reed, as the wind dies out, have become too feeble.

* The case is different for beats. If a major Third $\frac{5}{4}$ be sounded, the beats arising from the 4th partial of the lower and the 3rd of the upper note, relatively 16 and 15, are well heard separately, and their number is exactly $\frac{1}{4}$ of the number of vibrations made by the lower note in one second. But at the same time the differential tone arising from sounding 4 and 5 together is also heard, and it makes exactly the same number of vibrations in a second as the beats which are heard as separate sounds. I have heard this distinctly for the just major Third 256 : 320, beating 64 times in a second by its partials, and producing the differential tone 64, on Appunn's Tonometer. Dr. Preyer thinks that different parts of the ear hear the discontinuous beats and the continuous tone. But in the case of an original tone itself the separate beats are not heard. I believe the two phenomena to be totally distinct, and that beats and differential tones have no physical connection. They seem to me to have a different origin and to follow different laws.

This is not solved. Herr Appunn has made 31 tuning forks, forming just major scales from 11 C=2048 vib to 15 E−·139 or 183·863 sem=40960 vib. These were shown at the Loan Exhibition,* and I could hear them all distinctly, when carefully bowed, at the distance of 100 feet. Up to 14 C=16384 vib there was no difficulty at all, but 15 C, 15 D+·040, 15 E−·137 were not easy to hear. They were very distressing to many ears, though not to mine. Captain Douglas Galton exhibited a means of producing still higher tones, but with less power. For musical purposes tones higher than 12 C=4096 vib become useless, on account of the difficulty of recognising their intervals, and the mere mechanical tuning of such tones presents enormous difficulties.†

Hence the researches of Dr. Preyer tend to make the practical musical scale extend over 7 Octaves from 5 C to 12 C (32 to 4096 vib), though for the purpose of strengthening partials, as in organ mixture stops, 9 Octaves from 4 C to 13 C (16 to 8192 vib) may be available—certainly no more.

The next question is, *How accurately can we estimate pitch by the ear only?* Which is quite different from the question, *How accurately can we measure pitch?* The estimation of pitch demands a musical ear. The measurement of pitch is purely mechanical, and requires simply the power of hearing and counting beats.

The best means of measuring sensitiveness is difficult to determine. But without declaring it to be, *for a given pitch*, inversely proportional to the smallest number of mils perceived or the largest number of mils *not* perceived, it is evident that either set of numbers will serve as a very convenient scale for the purpose of comparing the sensitiveness of different persons. Observe that I have inserted the words 'for a given pitch,' for the sensitiveness of every ear seems to vary with the pitch of the note under observation.

To examine the sensitiveness of the ear for pitch, we must sound two notes in rapid succession, with either the same or different pitch, the exact difference having been first determined mechanically, and must take care that the quality of tone, loudness, duration, and steadiness of tone, should be as nearly as possible the same in each case. An alteration in any one of these four points confuses the ear. Then arise two questions—first, *Are the tones different?* secondly, *If so, which is the sharper?* These two questions are entirely different. Many ears will answer the first correctly which will not answer the second correctly. To prevent any influence on the judgment of the ear, no person should be present except the examiner and examinee, and the latter should not see what keys or valves are touched to produce the sound. Every precaution has to be taken, also, to have the ears and the whole nervous condition fresh and unwearied.

* These forks were also purchased by the South Kensington Museum at the close of the exhibition.

† Herr Appunn informed me that a hundred guineas would not pay him for the mere labour of making these 31 forks mentioned in the text.

To try his experiments Dr. Preyer had two instruments constructed by Herr Appunn, a Tonometer and a Differential Apparatus. The Tonometer contained 33 tones, from 7 C = 128 vib to 8 C = 256 vib, proceeding by differences of 4 vibrations, so that any two adjacent tones beat with each other 4 times in a second. But as this would have given a succession of perfect consonances, Dr. Preyer slightly altered the pitches, and carefully measured the result by counting the beats. The exact pitch of each note is therefore given in Table VI. in vib, and the sem between each and the lowest is annexed. The Differential Apparatus had 25 tones, from 500 to 501 vib proceeding by tenths of a vibration, and then 504, 508, 512, 1000, 1000·2, 1000·4, 1000·6, 1000·8, 1001, 1008, 1016, 1024, 2048, 4096 vib respectively.* Both were constructed with harmonium reeds and a wind regulator, to give a tone which could be depended on, and both were constantly proved by counting beats. With these, including Delezenne's results, Dr. Preyer found (p. 31) that—

at vib	or about	a difference of	giving an interval of	
119·791	6 B	·418 vib	60 mils	*was* perceived.
439·636	8 A	·364 „	14 „	
500	9 C	·300 „	10 „	
1000	10 C	·500 „	9 „	

But,

at vib	or about	a difference of	giving an interval of	
59·895	5 B	·209 vib	60 mils	was *not* perceived.
109·909	6 A	·091 „	14 „	
250	8 C	·150 „	10 „	
400	8 G♯	·200 „	9 „	

The intervals in the two cases are the same, but the pitch in the latter case was about an Octave lower than in the former, and hence the difference in the numbers of vibrations for the same interval is very different.

Combining all the results, we may state generally that throughout the scale $\frac{1}{5}$ vib is *not* heard;

but from 5 G♯ to 8 G♯, $\frac{2}{5}$ vib
and „ 8 A „ 9 C, $\frac{1}{3}$ „ } *is* heard.
„ „ 7 C „ 10 C, $\frac{1}{2}$ „

These differences, however, give very different intervals. Thus

$\frac{1}{5}$ vib	= 215 mils at	4 C	= 16	vib.
„	= 107 „	5 C	= 32	„
„	= 54 „	6 C	= 64	„
„	= 27 „	7 C	= 128	„
„	= 14 „	8 C	= 256	„
$\frac{1}{3}$ „	= 13 „	8 A	= 430·54	„
„	= 11 „	9 C	= 512	„
$\frac{2}{5}$ „	= 136 „	5 G♯	= 50·80	„
„	= 17 „	8 G♯	= 406·37	„

On examining the smallest intervals which could occur in just

* The Tonometer with 33 reeds (proper tune and not altered as for Dr. Preyer's purposes), together with another with 65 reeds (C 256 to C 512 proceeding by 4 vibrations at a time), was exhibited at the Loan Collection of Scientific Apparatus, and both were purchased at the close of the exhibition by the South Kensington Museum. The Differential Apparatus was not exhibited.

intonation, from the great Limma $\frac{27}{25} = 1\frac{8}{100}$, or 1·332 sem, to the skhisma $\frac{32805}{32768}$ or 20 mils, we see that it is only the last which runs any danger of not being heard by good ears. When the number of vibrations is less than 220, that is, when a note is below 7A, and two tones are sounded which are but a skhisma apart, they will differ only by $\frac{1}{4}$ vib, and hence the best ears may fail to detect the difference. But from 440 vib, that is from 8 A upwards, the difference will be $\frac{1}{2}$ vib or more, and will be always detected. The feeling of a singer, however, will allow him, even in the 7th octave, to take a Fifth with such precision, that, on sounding a tempered and just Fifth with his note, beats will be immediately heard in the first case and none in the second. I had an opportunity of witnessing this at a lecture given by Dr. Stone, at the South Kensington Museum, on July 25, 1876. This is, however, a different case. The same singer might not have detected that two notes, differing by a skhisma and sounded in succession at the pitch of the higher of his two notes, actually differed in pitch. In harmony, the beats produced by a skhisma are very distinct above 8 C, and without them we could hardly hope for equal temperament at all.

The above results refer principally to tones between 7 C and 10 C. Beyond these limits the sensitiveness of the ear decreases materially. Below 5 E = 40·32 vib practised ears will not recognise a whole vibration (which even at 5 E amounts to 424 mils), and unpractised ears will not perceive 2 or 3 vib (the first gives 839 mils, and the second 1·242 sem at 5 E). Above 10 C the sensitiveness probably diminishes, and after 11C the judgment becomes very uncertain. Unpractised ears, including those of pianists, did not distinguish even a minor Third in the 12th octave, although the difference was more than 1000 vib. Much of this want of sensitiveness, however, depends upon want of practice.

But it is also very singular that many of those who can tell that the pitch is different in the two cases cannot say with certainty which note is the sharper when the interval is small. Out of 398 trials, in 11 cases the ear could not say which tone was sharper, the difference amounting to $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{9}{10}$ vibration out of 1000 or 500 (giving in the former case about 7, 10, and 15 mils, and in the latter 14, 20, and 30 mils respectively). But these 11 observers were better off than 154 others, who pronounced the flatter tone to be the sharper, or conversely. Even in the case of 1000 and 1001 vib (17 mils at 10 C − ·41), the very best observers have sometimes mistaken the sharper for the flatter tone. (See P.S., p. 24.)

One practical result of this investigation is that, as already indicated, in comparing the relative sensitiveness of two ears, we must give not only the number of cents which they failed or succeeded in distinguishing, but also the pitch, or at least the approximate pitch, of the tone on which they were tried. It will be quite enough to give this pitch to the nearest whole number of sem, which can always be determined with sufficient exactness by a common harmonium or pianoforte. We may say that *a good ear for unisons* hears an error:

100	mils at	6C=	64	vib, and gradually	less on ascending.	
80	„	7C=	128	„	rapidly „	„
13	„	8C=	256	„	not so rapidly „	„
10	„	9C=	512	„	nearly the same	„
8	„	10C=	1024	„	not quite so little	„

For intervals greater than a unison it will be enough for musical purposes to note the semitone nearest to the *lower* note, and the amount of error heard or not heard in the *upper* note. In the following tables error will be always expressed in mils, and the marks ♯, ♭, placed before the number of mils respectively, will indicate that the upper note was too sharp or too flat by that amount. It does not always follow that the same amount of error was heard in both cases. The degrees by which the error was perceived will be distinguished by the expressions 'unheard, scarcely heard, just heard, heard, well heard, very well heard, very distinctly heard,' and the statements of observers, as given by Dr. Preyer, are interpreted by this scale. Sometimes the observer states whether the second note appears too sharp (♯) or too flat (♭). The principal opinions cited are those of Herr Georg Appunn, of Hanau, marked A, and those of Herr Michael von Davidoff, of Moscow, professional violinist, marked D, both men of a very great degree of sensitiveness of ear, highly cultivated by long practice. The letter U marks the judgments of other less practised ears. When no letter is annexed the observer is not named.

For the *Octave* the two first experiments are from Delezenne, the others from Dr. Preyer's own researches. In all the other intervals the Tonometer from 128 to 256 vib was used, carefully and constantly verified by counting the beats. All the lower tones, therefore, of the intervals are in the 7th Octave; hence the number 7 is omitted for convenience. The actual notes are indicated by the numbers explained in Table VI. I have given all the results contained in Dr. Preyer's pamphlet in this entirely new form, because, without examining them as a series, it is impossible to feel the great difficulties experienced in hearing an error, and still more in determining which direction it takes. Strange contradictions in this respect will be found in the minor Third, minor Sixth, and major Tone. The experiments are by no means sufficient for many intervals, especially for the Octave,*

* Mr. Herrman Smith writes to me that 'very few persons are able to discriminate between a perfect and a slightly imperfect interval of an Octave. It is only,' adds he, 'by interposing another interval that you can be certain of exactitude. I have often proved the best tuners at fault, to their great astonishment.' A mistuned Fourth above any note is a mistuned Fifth below the Octave of that note, and makes the same number of beats with each. Harmonium tuners, I believe, use this property to tune Octaves by. But this regards notes which are struck at the same time, not in succession, and merely shows the difficulty of observing the instant of the disappearance of beats, not the sensitiveness of the ear in recognising precision of interval when the notes are sounded successively.

Fifth, and major Sixth. It is to be hoped that Dr. Preyer may be able to complete them by a larger tonometer of two Octaves, 7 C to 9 C, which will enable the observer to take intervals exceeding an Octave, because all these are very interesting and important.

OCTAVE = 2 ÷ 1 vib = 12 sem.

Actual Notes (*see* Table VI.)	Approximate Lower Note	Precise Error in Mils	Statements of Observers
	6 F♯	♯ 34	unheard (Delezenne)
	6 F♯	♯ 69	just heard (Delezenne)
	8 C	♯ 35	well heard
	9 C	♯ 17	very well heard
	10 C	♯ 8	very distinctly heard
0.32	7 C	♯ 7	unheard
0.31	7 C	♭ 271	well heard, U

FIFTH = 3 ÷ 2 vib = 7·0196 sem. 7th Octave.

4.22	D	♯ 1	unheard, A, D
0.16	C	♯ 7	unheard, A, D
2.19	C♯	♯ 12	unheard, A, D
8.28	E	♭ 4	unheard, A, D
6.25	D♯	♭ 17	unheard, D; a trifle ♭, A
10.31	F	♭ 20	unheard, A, D (= tempered Fifth)

FOURTH = 4 ÷ 3 vib = 4·9805 sem. 7th Octave.

16.32	G	♭ ½	unheard, D
13.28	F♯	♭ 3	unheard, D
4.16	D	♭ 6	unheard, D
10.24	F	♭ 7	unheard, D
14.29	F♯	♭ 96	unheard, A, D
11.25	F	♭ 102	heard, A, D
8.21	E	♭ 103	just heard, A, D
5.17	D♯	♭ 122	heard, D; a trace ♭, A
2.13	C♯	♭ 126	well heard, D, U
7.20	D♯	♯ 2	unheard, D
1.12	C♯	♯ 6	unheard, D
15.31	G	♯ 82	unheard, A, D; a trace too ♯, A (the tempered Fourth is ♯ 20)
12.27	F♯	♯ 95	unheard, D; extremely little wrong, A
9.23	E	♯ 100	unheard, D; a trace too ♯, A
6.19	D♯	♯ 103	scarcely heard, A, D
3.15	D	♯ 127	heard, A, D
14.30	F♯	♯ 186	too ♯, A; just heard, D
11.26	F	♯ 194	well heard, DAD♭
2.14	C♯	♯ 256	too ♯, D,
1.13	C♯	♯ 397	distinctly heard, D

MAJOR SIXTH = 5 ÷ 3 vib = 8·8436 sem. 7th Octave.

Actual Notes (*see* Table VI.)	Approximate Lower Note	Precise Error in Mils	Statements of Observers
0.21	C	♭ 83	heard, A, D; very wrong, A
3.26	D	♭ 96	just heard, D; too ♭, D
6.31	D♯	♭ 121	well heard, A; too ♭, A
5.30	D♯	♯ 79	heard, A; just heard, D
2.25	C♯	♯ 109	unheard, D; too ♯, A
6.32	D♯	♯ 157	too ♯, A (this is the tempered major Sixth)
3.27	D	♯ 195	well heard
0.22	C	♯ 230	well heard, even by U

MAJOR THIRD = 5 ÷ 4 vib = 3·8631 sem. 7th Octave.

Actual Notes	Approximate Lower Note	Precise Error in Mils	Statements of Observers
8.18	E	♭ 1	unheard, A
4.13	D	♭ 4	unheard, A
13.24	F♯	♭ 52	unheard, D; too ♭, A
17.29	G	♭ 71	unheard, D; too ♭, A
1. 9	C♯	♭ 87	heard, A, D; too ♭, A
5.14	D♯	♭ 106	heard, A, D; too ♭, A
14.25	F♯	♭ 147	well heard, A, D; too ♭, D
18.30	G♯	♭ 148	well heard, D
2.10	C♯	♭ 191	well heard, A; just heard, D; too ♭, D
6.15	D♯	♭ 204	very well heard
12.23	F♯	♯ 8	unheard, A
0. 8	C	♯ 20	unheard, A, D
19.32	G♯	♯ 45	heard, A, D; too ♯, A
15.27	G	♯ 60	well heard, A, D
11.22	F	♯ 79	well heard, A, D
7.17	D♯	♯ 98	well heard, A, D; too ♭, D
3.12	D	♯ 99	well heard, D
18.31	G♯	♯ 120	unheard, D; scarcely heard, A; a little ♭, A (the tempered major Third is ♯ 137)
10.21	F	♯ 169	just heard, D; somewhat ♭, D
0. 9	C	♯ 448	a ♯ major Third, D

MINOR THIRD = 6 ÷ 5 vib = 3·1564 sem. 7th Octave.

Actual Notes	Approximate Lower Note	Precise Error in Mils	Statements of Observers
8.16	E	♭ 12½	unheard, A, D
18.28	G♯	♭ 13	unheard, D
14.23	F♯	♭ 58	unheard, A
19.29	G♯	♭ 68	unheard, D
9.17	E	♭ 73	unheard, A
4.11	D	♭ 78	unheard, D; a trace too ♭, A
15.24	G	♭ 120	unheard, D; a very little too ♭, A
20.30	G♯	♭ 123	scarcely heard, A; too ♭, D
10.18	F	♭ 140	heard, D; too ♭, A (the tempered minor Third is ♭, 156)
5.12	D♯	♭ 172	unheard, D; too ♭, A
16.25	G	♭ 174	unheard; scarcely heard, D a little too ♭; too ♭, A
21.31	A	♭ 189	heard, D
11.19	F	♭ 199	just heard, D
17.26	G	♭ 245	heard, D
1. 7	C♯	♭ 247	too ♭, D

MINOR THIRD = 6 ÷ 5 vib = 3·1564 sem—*continued.*

Actual Notes (see Table VI.)	Approximate Lower Note	Precise Error in Mils	Statements of Observers
13.22	F♯	♯ 6	unheard, A, D
3.10	D	♯ 14	unheard, A, D
7.15	D♯	♯ 66	unheard, A; heard, D
12.21	F♯	♯ 83	unheard, A; heard, D
21.32	A	♯ 91	too ♯, A, D
2. 9	C♯	♯ 98	heard, A, D; too ♭, D
16.26	G	♯ 124	unheard, A; scarcely heard, D
6.14	D♯	♯ 130	heard, A, D
11.20	F	♯ 138	a little ♯, A; heard, A, D; too ♭, D
20.31	G♯	♯ 147	too ♯, A, D
15.25	G	♯ 187	much too ♯; too ♭, D
0. 7	C	♯ 287	well heard
	MINOR SIXTH = 8 ÷ 5 vib = 8·1369 sem.		7th Octave.
8.32	E	♭ 13	unheard, A, D
0.19	C	♭ 47	unheard, A, D
5.27	D♯	♭ 76	unheard, A
2.22	C♯	♭ 120	unheard, D; somewhat too ♭, A
7.30	D♯	♭ 121	unheard, D; somewhat too ♭, A (the tempered minor Sixth is ♭ 137)
4.25	D	♭ 180	unheard, D; too ♭, A; scarcely heard, D
1.20	C♯	♭ 245	heard, D
8.31	E	♭ 291	just heard, D
3.23	D	♭ 304	heard; too ♭, D
3.24	D	♯ 7	unheard, A, D
1.21	C♯	♯ 91	unheard, A, D; just heard, D
4.26	D	♯ 116	heard, D; too ♯, A
7.31	D♯	♯ 148	heard, D
2.23	C♯	♯ 198	heard, A, D; too ♯, A
0.20	C	♯ 289	well heard
	MAJOR TONE = 9 ÷ 8 vib = 2·0392 sem.		7th Octave.
8.13	E	♭ 11	unheard, A, D
24.31	A♯	♭ 13	unheard, D
1. 5	C♯	♭ 37	unheard (F); too ♭, D (the tempered major Tone is ♭ 39)
17.23	G	♭ 42	heard, A, D; too ♭, A
25.32	A♯	♭ 42	heard, A; too ♯, A
9.14	E	♭ 57	heard, D; unheard, A
18.24	G♯	♭ 82	heard, D
2. 6	C♯	♭ 90	heard, D
19.28	G♯	♭ 119	scarcely heard, D
11.16	F	♭ 144	heard, D
14.19	F♯	♭ 243	heard, D
16.22	G	♯ 8	unheard, A
23.30	A	♯ 28	unheard, A; a little too ♯, A; just heard, D
7.12	D♯	♯ 38	unheard (F); heard, D
15.21	G	♯ 56	heard, A; unheard, D
22.29	A	♯ 64	a little ♯, D
14.20	F♯	♯ 94	heard, D

The order in which the above results are given is not that which Dr. Preyer has adopted, nor exactly that in which he subsequently says that the consonances seem to arrange themselves in respect to sensitiveness of the ear, for I have reversed the order of the major Sixth and major Third. It will be seen that in no case is any error detected when it does not exceed a cent or at least 8 mils. This justifies us in using the convenient cent as the practical unit for small intervals. We have no proper indications of sensitiveness for the unison and Octave in the 7th Octave, and hence these are conjecturally placed in the following Table, which for the other intervals gives the actually smallest intervals heard, and actually greatest intervals unheard by any observer in the 7th Octave, as recorded in the preceding Tables, and then in the last column subjoins the equivalent of Dr. Preyer's mean. This mean does not represent the precise mean of the recorded observations, but is obviously founded upon other trials in addition, and is, perhaps, partly theoretical. This is acknowledged to be the case for the Fifth, and is certainly so also for the major Sixth. At present the jump from the Fifth to the Fourth is quite surprising:

Through the 7th Octave (128 to 256 vib).

For the Interval	The least ♯ or ♭ error heard, had cents	The greatest ♯ and ♭ error unheard, had cents.	Dr. Preyer's mean, has cents
Unison?	♯ 5	♯ $1\frac{1}{2}$	—
Octave?	♯ 5	♯ 2	—
Fifth	?	♯ 1, ♭ 2	4
Fourth	♯ 8, ♭ 10	♯ 10, ♭ 10	10
Major Sixth	♭ 8	♯ 11	12
Major Third	♯ 5, ♭ 5	♯ 12, ♭ 7	11
Minor Third	♯ 7, ♭ 8	♯ 12, ♭ 17	18
Minor Sixth	♯ 9, ♭ 12	♯ 9, ♭ 18	18
Major Tone	♯ 3, ♭ 4	♯ 4, ♭ 4	—

The great difference which practice produces in the judgment formed by the ear, leads me to suppose that its real sensitiveness depends not only upon original constitution of the ear, but, as in most other cases of nervous action, more especially on steady, continued exercise, and that consequently with the same original constitution those intervals most frequently used are most accurately gauged. Now of unisons in melodic succession we have comparatively little practice, except in tuning the piano, (and then the notes are often struck together,) for in playing music a unison is produced generally by repeating an excitement under the same circumstances without any judgment of the ear. I observed that the unison reeds on my English concertina were tuned as Fifths to the same note, and not as unisons to each other, and the tuner told me that he found that the easiest plan, although such reeds could be easily made to sound simultaneously, and thus to give beats. In the unison-strings of a pianoforte it seems to me that the tuner relies more on the Octave than on the unison itself, but here I may be wrong. The Octave when played or sung requires totally different preparation, and hence the attention of the performer is early and constantly directed to it. He is also constantly hearing it in the voices of men and women, or men and boys. Would equal sensitiveness be shown for the double Octave, which does not lie within the compass of perhaps the greater number of singer's voices? The Fifth is the very foundation of the musical scale, and to master it thoroughly is a necessity of life to the violinist. We should, therefore, expect that for all singers and violinists, and especially for such men as Herr Appunn and Herr Davidoff, the sensitiveness for the Fifth would be abnormally high. We see, indeed, by the tables, that it is widely distinguished from all other consonant intervals and is only approached by the dissonant interval of a Major tone. This, again, is merely an Octave lower than the double Fifth, and is more likely to be heard correctly than the semitone. In ordinary equal temperament it is only 4 cents too flat, and this is precisely the amount of error marked as heard and unheard for the interval. This leads to the conception that the habit of hearing equal temperament and of singing and playing to equally tempered instruments has probably much concern in determining the sensitiveness of the cultivated, as distinguished from the uncultivated but naturally sensitive ear. The other intervals, except the Fourth, are so materially affected in character by temperament, that we are even surprised to find any observers who detect the tempered Thirds and Sixths as false. Possibly singers brought up to sing just intervals only would feel the error more acutely. I have noticed that in trying to judge of a tone or interval given by any instrument, the observer almost mechanically sings it to himself, that is, translates it into the quality of tone with which he is most familiar, and he seems to me (I may be quite wrong), to found his judgment rather upon this imitation (which, of course, is liable to differences) than upon the original sounds. I would thus account for the seizure of intervals between simple tones, which in themselves have so slight a hold upon the internal ear, on account of the absence of upper partials.

The principle by which the consonant intervals are perceived, seems to me to be that conjectured by Helmholtz—the remembered identity of a partial tone in the second note, with a partial tone in the first (Helmholtz, 'Sens. of Tone,' p. 394). This conception has been somewhat differently carried out by Dr. Preyer (p. 60), who, reckoning each note as having 8, or 12, or 16, or 24 partials, determines how many partials would be identical in the two tones. But I think there is more than mere repetition. The tones compared have rarely more than 8 or 12 effective partial tones, and the higher of these are generally so high and weak, that they would probably not much affect the judgment of the ear. In the following Table each tone is supposed to have only 8 partial tones. For convenience, the 7 just tones $6c$, $7c$, $7g$, $8c$, $8e_1$, $8g$, $9c$,* are taken so that the numbers

* C, D, &c. are used for the equally tempered notes of the arithmetical

$6c$	$7c$	$7g$	$8c$	$8e_1$	$8g$	$9c$
1						
2	2					
3	...	3				
4	4	...	4			
5	...	...	...	5		
6	6	6	...	...	6	
7	...	...	...	...	...	
8	8	...	8	...	...	8
	...	9	...	...	...	...
	10	...	...	10	...	...
	12	12	12	...	12	...
	14	...	...	...	...	...
	...	15	...	15	...	...
	16	...	16	...	...	16
		18	...	...	18	...
		...	20	20	...	...
		21	...	...	...	...
		24	24	...	24	24
			...	25	...	...
			28	...	...	...
			...	30	30	...
			32	...	...	32
				35	...	..
				...	36	...
				40	...	40
					42	...
					48	48
						56
						64

assigned to the various partials, when multiplied by 64, will give their vib. Now observe that $7c$ contains exactly 4 partials (2, 4, 6, 8) the same as those of $6c$ the Octave below, but $7g$ contains only 2 partials (6, 12) the same as those of the Fifth below $7c$, and $8c$ also contains 2 partials (12, 24) the same as those of $7g$ the Fourth below. Hence this principle does not distinguish the Fifth and Fourth without taking more partials into consideration. Again, in all the other intervals the major Third $8c$ to $8e_1$, the major Sixth $7g$ to $8e_1$, the minor Third $8e_1$ to $8g$, and the minor Sixth $8e_1$ to $9c$, the upper tone has only 1 partial in common with the lower. It seems, therefore, necessary to be satisfied with one common partial only, and to suppose that the differences depend upon the position of that partial with respect to the others. Now the higher the partial the less distinctly it is perceived and the more it is veiled by the lower and louder partials below it. This is but an illustration of Prof. Alfred Mayer's recent investigations on the extinction or at least veiling of a higher weaker tone by a lower and stronger tone, while conversely a higher stronger tone does not extinguish or veil a lower weaker tone.* Now for the Octave $\frac{2}{1}$ the first identical partial is the lowest in the upper note and the lowest but one in the lower note. For the Fifth $\frac{3}{2}$ it is the 2nd of the upper which is identical with the 3rd of the lower note. The result ought to be more difficult to perceive than in the former case, and I cannot doubt that more special practice is generally necessary to recognise a Fifth than an Octave. I recommend this point to the attention of teachers of beginners in naming intervals heard, and distinguishing whether the second note is too sharp or too flat. For the Fourth $\frac{4}{3}$ it is the 3rd partial of the upper tone, which is identical with

scale, where 0 C marks 1 vibration in a second; C′, D′, &c., are used for the equally tempered notes when the initial pitch 0 C′ is not 1. Similarly the small letters *c*, *d*, &c., with the additional conventions shown in Table V., represent just tones, beginning with 0 C and *c*′, *d*′, &c., with the same additional conventions, the just tones commencing with 0 C′.

* See his paper in *Nature*, for 10th Aug. 1876, p. 318.

the 4th of the lower, and this must require very much more attention and care. The Fourth is also an interval much more seldom practised now, as we do not usually tune by it, so that it is best known as the inversion Fifth, and not as a substantive interval. Possibly when the old Greek Tetrachord was founded on the Fourth, the ear was practised into much greater sensitiveness for that interval. In case of both the major Sixth $\frac{5}{3}$ and major Third $\frac{5}{4}$, the common partial is the 5th partial of the lower note, but it is the 3rd partial of the upper note for the major Sixth, and as high as the 4th partial of the upper note for the major Third; this would certainly cause it to be best perceived for the major Sixth, and best remembered; so that we should naturally expect the ear to be more sensitive for errors in the major Sixth than in the major Third. But the major Sixth is the worst tuned of all the intervals in the equal temperament, being 16 cents too sharp, and it is of much less frequent occurrence than the major Third, hence there is a possibility of the latter being better heard, notwithstanding that it is 14 cents too sharp in the equal temperament. On account of the few examples of mistuned major Sixths, which Dr. Preyer could produce on his altered tonometer, it is not possible to decide between the cultivated sensitiveness of the ear for these two intervals. For the minor Third $\frac{6}{5}$ and minor Sixth $\frac{8}{5}$ the common partial lies so far up in each case that it is only surprising that so much sensitiveness was met with. In the examples of minor Thirds and minor Sixths, obtained by Messrs. Cornu and Mercadier (see my translation of Helmholtz, p. 791), they were always taken too flat. For the minor Third their amateur violinists made the upper note from 13 to 35 cents too flat, and a professional violoncellist made it 5 to 21 cents too flat. For the minor Sixth the amateurs were from 12 to 33 cents flat, and the professional violoncellist once only 4 cents, but three times 20 cents too flat. These serve to show how very imperfectly fixed this interval is in the minds of performers.

If we carried out this principle for the ordering of intervals according to sensitiveness, including intervals of more than one but less than two octaves, we ought to arrive at the following natural order, which, however, would be materially affected by practice. Instead of using fractions, as $\frac{3}{2}$, $\frac{5}{4}$, &c. to express the interval, the equivalent expressions $3 \div 2$, $5 \div 4$, &c. are employed, as they are easier to see, and as these numbers show the order of the common partials, the higher number giving the partial in the lower note, it is important to see them at a glance. These are placed in the column headed 'vib.' The name of the interval and an example occupy the last column. The cross line separates consonant from dissonant intervals. This would of course not necessarily be the order of the comparative value consonances and dissonance, the feeling for which depends on other matters.

Vib	Sem	Name
1 ÷ 1	0	Unison, 8*c* to 8*c*
2 ÷ 1	12	Octave, 8*c* to 9*c*
3 ÷ 1	19·0196	Twelfth, 8*c* to 9*g*
3 ÷ 2	7·0196	Fifth, 8*c* to 8*g*
4 ÷ 3	4·9805	Fourth, 8*g* to 9*c*
5 ÷ 2	15·8631	major Tenth, 8*c* to 9e_1
5 ÷ 3	8·8436	major Sixth, 7*g* to 8e_1
5 ÷ 4	3·8631	major Third, 8*c* to 8e_1
6 ÷ 5	3·1564	minor Third, 8e_1 to 8*g*
8 ÷ 3	16·9805	Eleventh, 7*g* to 9*c*
8 ÷ 5	8·1469	minor Sixth, 8e_1 to 9*c*
9 ÷ 4	14·0391	major Ninth, 8*c* to 9*d*
9 ÷ 5	10·1760	large minor Seventh, 8e_1 to 9*d*
9 ÷ 8	2·0391	major Tone, 8*c* to 9*d*
10 ÷ 3	20·8436	major Thirteenth, 7*g* to 9e_1
10 ÷ 9	1·8240	minor Tone, 8e_1 to 8*d*
12 ÷ 5	15·1564	minor Thirteenth, 8e_1 to 9*g*
15 ÷ 4	22·8827	major Fourteenth, 8*c* to 9*b*
15 ÷ 8	10·8827	major Seventh, 8*c* to 8*b*
16 ÷ 5	20·1469	minor Thirteenth, 8e_1 to 10*c*
16 ÷ 9	9·9609	small minor Seventh, 8*d* to 9*c*
16 ÷ 15	1·1173	diatonic Semitone, 8e_1 to 8*f*

There are at least four popular ways of teaching to sing: first, by the feeling for intervals, between two successive notes, which is Mr. Hullah's plan; next, by the feeling for diatonic succession only, the successive notes being mentally connected by such a chain, which is M. Chevé's plan; thirdly, by the mental effect of each tone in the major and relative minor scales, which is the Tonic Solfa plan; and last of all, most general I fear, by playing the air on a piano, and imitating it. The extreme variety of intervals in just intonation confines the first plan to tempered intonation, and yet it is only in just intonation that the principle of an identified partial, giving the true feeling of an interval, is possible. The second plan was originally developed for a tempered scale, differing from the usual equal temperament,* and it is hardly applicable to just intonation. The third method was suggested by the practice of just intonation, to which it appears exactly suited. The fourth plan reduces everything to learning 'by ear.' There is a fifth plan possible for those who can remember every tone within the compass of their voice. Determining what is the precise interval heard, and proceeding by a known interval from one tone to another, are two different things, and probably the second is much easier than the first. Dr. Preyer is, however, dealing only with the first.

* Galin, the introducer of the method, advocated Huyghens' cycle of 31 notes to the Octave, with nearly perfect major Thirds and flat Fifths (*Exposition d'une Nouvelle Methode pour l'Enseignement de la Musique*, 1816; reprinted 1862, p. 162). Émile Chevé, its chief propagator, uses the cycle of 29 to the Octave, which gives sharper major Thirds than the Greek, and also sharp Fifths, of which he was apparently not aware, for no one could have endured them (*Méthode Élémentaire de Musique Vocale*; this division results from p. 292).

For the 6th Octave the sensitiveness is less than the 7th, which is that to which all the above experiments on intervals relate. Below the 6th Octave, sensitiveness decidedly diminishes. In the 8th and 9th Octave sensitiveness increases, but in the 10th it decreases, and from the 11th upwards it is very small indeed, one reason being, of course, that the ear is little practised in estimating such sounds with accuracy. The following results actually obtained by observation show how exceedingly uncertain are the judgments of the ear, on such unusual positions.

In the first list referring to the 4th and 5th Octaves, only erroneous judgments are given; the correct opinions were, however, more numerous than the incorrect.

Lower Tone in 4th Octave, 16 to 32 vib.

Vib	Sem to cents	Real Name of Interval	Name assigned to the Interval	Error in sem to cent
32÷16	12	Octave	about a Fourth	about ♭ 7·02
48÷16	19·02	Twelfth	doubtful	?
36÷18	12	Octave	doubtful, Octave	?
38÷19	12	Octave	Octave of the Fourth	♯ 4·98
30÷20	7·02	Fifth	doubtful, Octave	?
50÷25	12	Octave	major Sixth	♭ 3·16
50÷29	9·43	59 cents sharper than a major Sixth	minor Sixth	♭ 1·29
50÷30	8·84	major Sixth	minor Sixth	♭ ·70
		Lower Tone in 5th Octave, 32 to 64 vib.		
64÷32	12	Octave	doubtful	?
40÷36	1·82	minor Tone	major Tone	♯ ·22
128÷56	14·31	27 cents sharper than a major Ninth	major Ninth	♭ ·22

In the next table the opinions of four of Dr. Preyer's best observers, distinguished as A, D, N, and Z, alone are given, and they possess much interest as being the judgments formed by excellent musicians, on hearing such tones for the first time. The intervals were all just. The trials were made with König's rods and Appunn's forks.

The Lower Tone in the 12th Octave, 4096 to 8192 vib.

Vib	Sem to cents	Real name of Interval	Names assigned to the Interval by A	D	N	Z
5120÷ 4096	3·86	ma. 3rd	5th	4th	major 2nd	—
6144÷ 4096	7·02	5th	3rd, 5th	?, 5th	Tone, mi. 3rd	?, major 2nd
8192 ÷ 4096	12	Octave	—	Octave	4th	major 3rd
6144÷ 5120	3·16	mi. 3rd	—	? (4th)	nearly Unison	—
8192÷ 5120	8·14	mi. 6th	5th	?	—	—
,, ,,	,,	,,	—	?	?	—
10240÷ 5120	12	Octave	—	5th, Oct.	—	7th
8192÷ 6144	4·98	4th	3rd	5th, Oct.	Tone	—
12288÷ 6144	12	Octave	Octave	5th	—	—

The Lower Tone in the 13th Octave, 8192 to 16384 vib

Vib	Sem to cents	Real name of Interval	Names assigned to the Interval by A	D	N	Z
8192 ÷ 8192	0	Unison	—	Unison	—	—
10240 ÷ 8192	3·86	ma. 3rd	6th	?	?	—
" "	"	"	—	5th, ?	—	—
12288 ÷ 8192	7·02	5th	—	6th	—	4th; and 5th
" "	"	"	—	?	3rd	—
16384 ÷ 8192	12	Octave	Octave	Octave, ?	—	—
12288 ÷ 10240	3·16	mi. 3rd	5th	—	—	4th, ma. 3rd, 3rd
16384 ÷ 10240	8·4	mi. 6th	—	ma. 6th	—	—
20480 ÷ 10240	12	Octave	—	?	?	—
16384 ÷ 12288	4·98	4th	4th	Unison	—	2nd, 4th
20480 ÷ 12288	8·84	ma. 6th	—	6th	?	—

This table, however, does not represent the full amount of confusion. From 7 C=128 vib, to 10 C=1024 vib, the four observers never once stated the intervals incorrectly. The observers were often in doubt, shown by (?) in the above table. A never doubted, D doubted 35·4 times per cent, N 36·3 times per cent, and Z 7·1 times per cent. But when they did not doubt, A was wrong 54·5 times per cent, D 42·3 times per cent, N 63·6 times per cent, and Z 64·2 times per cent. Or taking the actual numbers, out of 48 cases where no doubt was expressed, 31 judgments were wrong (or 64·5 per cent). Of these 31 errors, 20 gave the interval too small, and 11 too large.

Dr. Preyer deduces from this investigation, that a 'physiological temperament' would bear to have greater errors below 6 C and above 11 C, than elsewhere. It is evident, however, that no treatment would be useful for harmonic purposes, in which the Octave was not treated in the same way throughout. That such a proposal could be made, shows the fundamental differences between melody, or successions of tones, of which no two are sounded together, and harmony, or the simultaneous sounding of two or more tones. The question of temperament, therefore, depends upon a multitude of other considerations, which do not come within the scope of this paper. But it is evident that, when an ear is constantly exercised in hearing and using any temperament, its sensitiveness for consonant intervals must necessarily be impaired. And, from experiment made on the differences between the harmonic effects of a major Tone $\frac{9}{8}$, a minor Tone $\frac{10}{9}$, and even a diatonic semitone $\frac{16}{15}$, when taken just, or slightly mistuned, I am led to think that the same is likely to be true for dissonant intervals.

To sum up by a single question, *What is a good ear for Music*, considered merely in its melodic relations? We may, perhaps, give the following partial answer.* A good ear is one which,

* The limitation 'partial' must not be overlooked. As far as my own opinion goes, a very great deal more is required. But here it was necessary to confine myself to the matter brought out in this paper.

within the 6th, 7th, 8th, and 9th Octaves appreciates, both in existence and direction, an interval of one or two cents in Unisons, Octaves, and Fifths, and ten to fifteen cents in other intervals. Such an ear must in the 8th and 9th Octaves appreciate that an equally tempered Fifth is flat, and an equally tempered Fourth is sharp, and in all the Octaves that the equally tempered major Sixth and major Third are decidedly much too sharp, and, perhaps, but not so certainly, that the equally tempered minor Third and minor Sixth are too flat. As regards the equally tempered major Seventh, there is such a habit of using a still sharper tone for the 'leading note,' that no ear of singer or violinist can be trusted for it.* A good ear ought also in these Octaves to distinguish a minor from a major Tone.

In conclusion, I would observe that I did not answer the question by saying 'a good ear knows when a note is in tune,' because the meaning of 'being in tune' is at present unfixed both as to standard pitch and desired interval.

* In Messrs. Cornu and Mercadier's experiments, cited in my translation of Helmholtz, there are very few specimens of major Sevenths, but the four instances by amateurs are 21, 17, 32, 29 cents too sharp, and the four of the professional violinists are 34, 40 (thrice) cents too sharp as compared with the tone which makes a just major Third with the Fifth of the scale, or $b = 10{\cdot}88$ sem; the actual pitches were 11·09, 11·05 11·20, 11·17 (the 61st harmonic duly reduced) for amateurs, and 11·22, 11·28 (thrice) for professionals. This is almost as bad as the sharp F′ (11th harmonic, 53 cents too sharp) on the trumpet, and in all chords of the dominant in which it occurred the effect would be horrific. But it is evident that ears attuned to such intervals are likely to be offended at just major Sevenths intended for harmony, and still more at the major Sevenths of the mean tone temperament which are a quarter of a comma or 5½ cents flatter still, as I had occasion to remark when Mr. Bosanquet played a specimen of mean tone temperament (the only one used by Handel) before the Musical Association. (Proceedings for 1874–5, p. 128, note.)

P.S.—To show the difficulty of appreciating slightly disturbed unisons, as mentioned on p. 12, I exhibited two forks and asked for opinions as to which was the sharper. Dr. Stainer, Dr. Stone, and many others were of one opinion, and Mr. W. H. Cummings and many others held the contrary opinion. In my hurry I said that the first were wrong and the second right. (See the discussion on p. 30.) But on subsequently very carefully proving the forks I found that I was in error, and that Dr. Stainer and Dr. Stone were right, and I communicated the result of my proofs of the forks to the Association, through Dr. Stone, at the next meeting. The absolute pitches, by Appunn's tonometer, were, 529·3 and 529·5, as nearly as I could determine. That is, the difference was at the limit of the power of the ear to determine (p. 11). I tried the same two forks on another occasion before very sensitive ears, with a similar division of opinion. Some other excellent ears, which finally decided correctly, were a long while before they could come to a conclusion. I decided by slightly warming each fork alternately under my arm, which flattened its pitch, and then observing whether the beats were increased or diminished in frequency, as proposed on p. 2. A. J. E.

Table 1.

Division of the Octave into Sem, Tithes, Cents & Mils.

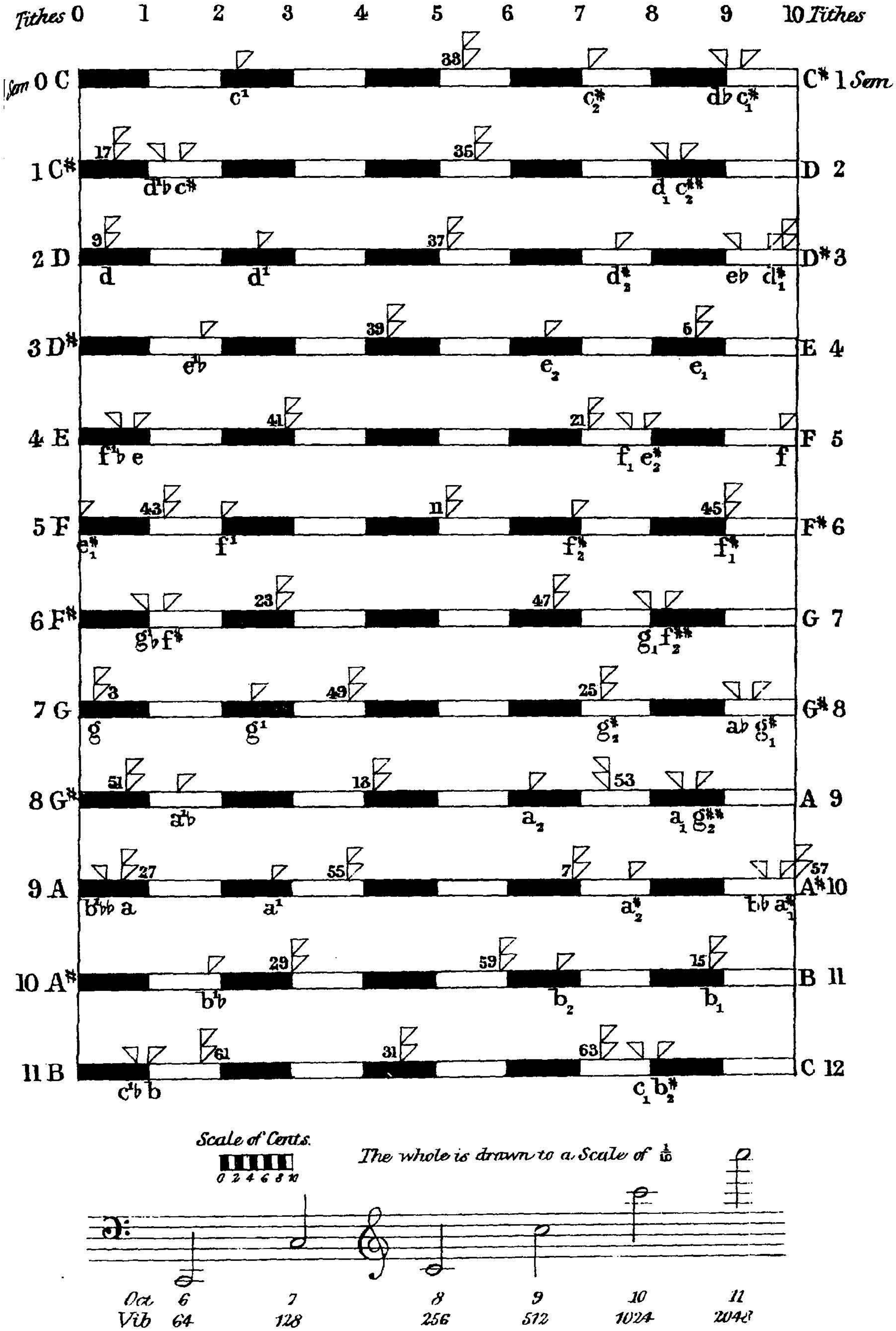

Table II.

CALCULATION OF INTERVALS AS SEM, TITHE, CENT, MIL, AND DIME.

A. Octave			B. Sem		
Note	Sem	Vib	Note	Sem	Quotient
0 C	0	1	C	0	1,000,000
1 C	12	2	C♯	1	1,059,462
2 C	24	4	D	2	1,122,462
3 C	36	8	D♯	3	1,189,207
4 C	48	16	E	4	1,259,921
5 C	60	32	F	5	1,334,840
6 C	72	64	F♯	6	1,414,213
7 C	84	128	G	7	1,498,307
8 C	96	256	G♯	8	1,587,401
9 C	108	512	A	9	1,681,793
10 C	120	1,024	A♯	10	1,781,797
11 C	132	2,048	B	11	1,887,748
12 C	144	4,096			
13 C	156	8,192			
14 C	168	16,384			
15 C	180	32,768			
16 C	192	65,536			

C. Tithe		D. Cent		E. Mil		F. Dime	
Tithe	Quotient	Cent	Quotient	Mil	Difference	Dime	Difference
0	1,000,000	0	1,000,000	0	0	0	0
1	1,005,793	1	1,000,578	1	58	1	6
2	1,011,619	2	1,001,156	2	116	2	12
3	1,017,480	3	1,001,734	3	173	3	17
4	1,023,374	4	1,002,313	4	231	4	23
5	1,029,302	5	1,002,892	5	289	5	29
6	1,035,265	6	1,003,472	6	347	6	35
7	1,041,262	7	1,004,052	7	405	7	41
8	1,047,294	8	1,004,632	8	463	8	46
9	1,053,361	9	1,005,212	9	521	9	52

Directions for using Table II.

1. *Calculation of a Pitch to Dimes.*—Given any pitch as 446·57; look in A: for the next least vib, 256; this determines that the note lies in the Octave begun with 8 C, or 96 sem.

Multiply the pitch by 1,000,000, giving 446,570,000, and divide by the 256 just found, to the nearest unit. That is, if the last remainder is less than half the divisor, neglect it; if as much, or more, increase the quotient by one. This quotient will always have 7 figures, and in this case is 1,744,410.

Look in B. for the next least quotient, 1,681,793, giving 9 sem. Multiply the previous quotient by 1,000,000, and divide the result by this next least quotient to the nearest unit. Then 1,744,410,000,000 divided by 1,681,793 gives 1,037,238 as quotient.

Look in C. for the next least quotient, 1,035,265, corresponding to 6 tithes. Multiply the greater quotient by 1,000,000 and divide by the lesser to the nearest unit. Then 1,037,238,000,000 divided by 1,035,265 gives 1,001,906 as quotient.

Look in D. for the next least quotient, 1,001,734, giving 3 cents. Subtract this from the last quotient, giving remainder 172.

Look in E. for the next least remainder, or a larger remainder, not differing by more than 3. We find 173 giving 3 mils, and as this is in excess, but does not differ by so much as 3, there are no dimes. If it had differed by 3 or more, we should have reckoned one dime less.

Result, 446·57 vib, gives 8 Octaves, 9 sem, 6 tithes, 3 cents, 3 mils, and no dimes; or, since 8 Octaves = 96 sem, it gives 105·6330 sem.

2. *Calculation of Pitch to Cents.*—If the result is only wanted to the nearest cent, much labour may be saved by multiplying at first by 1,000 only, for B. and C.: that is, omitting the three last figures in the quotients of B. and C., and then multiplying by 10,000: that is, omitting the two last figures in D., taking care always to increase the last figure retained by 1, if the first figure omitted is 5 or more. Thus in the last case, dividing 4,465,700 by 256, we have 1,744, to which 1,682, giving 6 sem, is nearest in B. Dividing 1,744,000 by 1,682, we have 1,037, to which 1,035, giving 3 tithes, is nearest in C. And dividing 1,037,000,0 by 1,035, we have 1,001,9, to which 1,001,7 is nearest in D, giving 3 cents. Hence the result is 105·63 sem, which is correct.

3. *Calculation of an Interval.*—If an interval is given as a fraction, as 15 ÷ 2, first multiply the smaller number as often by 2 as is necessary to make the fraction less than 2, and for each such multiplication add 12 sem. Here we must multiply twice by 2, giving 15 ÷ 8, and shall have to add 24 sem. Now proceeding for cents only, which is generally sufficient, 15,000 ÷ 8 gives 1875, the nearest to which in B. is 1,782 (last figure increased by 1) giving 10 sem. Then 1,875,000 ÷ 1,782 gives 1,052, the nearest to which in C. is 1,047, giving 8 tithes. And 1,052,0000 ÷ 1,047 gives 1,004,8, the nearest to which in D is 1,004,6, giving 8 cents. Result, 34·88 sem. This process is even more rapid than by a table of logarithms.

4. *Calculation of a Pitch or Interval from Cents.*—The inverse process, to find the pitch or interval from the sem and cents, is a mere inversion of the preceding. Thus, to find the fraction expressing the interval 10·88. By D., 8 cents gives 1,004,6, and by C., 8 tithes give 1,047. Multiply, product 10,518,162, from which the four last figures, 8,162, have to be omitted, and since the first of these is 8, we must increase the one before it by 1, giving 1,052. By C. the 10 sem give 1,782, and multiplying this by 1,052 we have 1,874,664, from which the three last figures, 664, have to be omitted, giving 1,875. Dividing this by 1,000 we have 1·875, which is the expression of $\frac{15}{8}$ as a decimal fraction.

5. *Calculation of all the usual Intervals.*—All the intervals commonly used in music can be calculated by Table IV. to dimes. Thus, opposite 15 is 46·8827 and opposite 8 is 36; on subtracting we find that $\frac{15}{8}$ gives 10·8827 sem. Similarly for 81 ÷ 80, since 81 = 9 × 9, we must double what is opposite 9, or 38·0391: the result is 76·0782. Then 80 is 5 × 16; opposite 5 is 27·8631, and opposite 16 is 48, the sum being 75·8631. Subtracting this from 76·0782, the number found for 81, we have ·2151 sem. The preceding processes (1, 2, 3) need therefore only be used for unusual combinations in which the numbers are not the product of others less than 64.

For more exact processes see my translation of Helmholtz's *Sensations of Tone*, Appendix XIX., Table I., pp. 743–5.

Table III.

TO FIND THE INITIAL NUMBER OF VIBRATIONS FOR ANY STANDARD PITCH.

Division A.— The column 8 A′ contains the extreme range of the values of 8 A′ in use, proceeding by four vibrations at a time. The columns 9 C′ and 0 C′ give the corresponding values of 9 C′ and 0 C′, and the column 'mils' the number of mils by which the given standard is flatter or sharper than the Arithmetical Standard. For intermediate values multiply the number in the columns D by the difference between the value of 8 A′, given, and the next least value of 8 A′ in the table. Thus, if 8 A′ = 437·27, the next least value in the table is 436, and difference 1·27. The products of this

1·27 by 1·19, ·00232, and 40 (the numbers in the three columns D), are 1·51, ·00295, and 51, and these results added to 518·49, 1·01268, 219♯; which opposite to 436 give 520, 1·01563, and 270, which agree with the values opposite 9 C′ = 520 in Division B.

Division B.—The column 9 C′ contains the extreme range of the values of 9 C′ in use, proceeding by four vibrations at a time. The other columns give the corresponding values of 8 A′, 0 C′, and mils sharper or flatter than arithmetical pitch as before. Intermediate values found as in Division A.

Note.—The results of these divisions, A and B, can be obtained directly thus:—Multiply 8 A′ by 1·1892 to find the corresponding equally-tempered 9 C′. Multiply 9 C′ by ·8409 to find the corresponding equally-tempered 8 A′. Divide 8 A′ by 8 A = 430·539, or 9 C′ by 9 C = 512, to find 0 C′. From 0 C′ find the mils by Table II.

Division C.—It is not easy to determine the exact vibrations of any given note purporting to be 8 A′ or 9 C′. Until Appunn's Tonometer was exhibited in the Loan Collection of Scientific Apparatus there was no instrument in England by which it could be done accurately. The standard French 8 A′, or Diapason Normal, intended to be 435, is really 439, and this has been the cause of many errors. The Society of Arts 9 C′, intended to be 528, was really 539·4. The fork originally published for Mr. Hullah, intended for 9 C′ = 512, is really 529⅓. The fork published for Mr. Curwen, also intended for 9 C′ = 512, was 517, but has recently been corrected. The results I obtained by Appunn's Tonometer were published in the *Athenæum* for 2nd and 30th Dec. 1876, pp. 731 and 893, subsequently to the reading of this paper, although some of them were mentioned to the Musical Association at the time. These are incorporated with many other interesting results in a paper read by me before the Society of Arts on 23rd May 1877. In preparing this table for the press I have thought it best to add some of the most interesting of those results in Division C, arranged as in other divisions.

DIVISION A.

8 A′	9 C′	D	0 C′	D	Mils	D
424	504·22	1·19	0·98481	·00232	265 ♭	41
428	508·98	1·19	0·99410	·00232	102 ♭	40
432	513·73	1·19	1·00339	·00232	59 ♯	40
436	518·49	1·19	1·01268	·00232	219 ♯	39
440	523·25	1·19	1·02197	·00232	376 ♯	39
444	528·00	1·19	1·03127	·00232	533 ♯	39
448	532·76	1·19	1·04055	·00232	688 ♯	39
452	537·52	1·19	1·04984	·00232	842 ♯	38
456	542·28	1·19	1·05914	·00232	995 ♯	38
460	547·03		1·06843		1146 ♯	

DIVISION B.

8 A′	D	9 C′	0 C′	D	Mils	D
423·81	·84	504	0·98438	·00195	274 ♭	34
427·18	·84	508	0·99219	·00195	136 ♭	34
430·54	·84	512	1·00000	·00195	000 ♯	34
433·90	·84	516	1·00781	·00195	135 ♯	33
437·27	·84	520	1·01563	·00195	269 ♯	33
440·63	·84	524	1·02344	·00195	401 ♯	33
444·00	·84	528	1·03125	·00195	533 ♯	33
447·36	·84	532	1·03906	·00195	664 ♯	32
450·72	·84	536	1·04688	·00195	793 ♯	32
454·09	·84	540	1·05469	·00195	922 ♯	32
457·45	·84	544	1·06250	·00195	1049 ♯	32
460·82		548	1·07042		1178 ♯	

Division C.

8 A′	9 C′	C′	Mils	Remarks
426·4	507·14	·99051	165 ♭	Handel's own fork
430·539	512	1·00000	0 ♯	Arithmetical Pitch
434·75	517	1·00976	168 ♯	Messrs. Curwen's and J. H. Griesbach's, 512
435	517·3	1·01036	178 ♯	Theoretical French, 8 A′
439	512·06	1·01966	337 ♯	Actual French, 8 A′, *Diapason Normal*
439·79	523	1·02148	367 ♯	Broadwood's 'Low Pitch'
440	523·25	1·02197	376 ♯	Scheibler's or Stuttgart Pitch
444	528	1·03125	532 ♯	Theoretical 9 C′ of Society of Arts
445·09	529·3	1·03364	575 ♯	Mr. Hullah's 8 C, meant for 512.
449·88	535	1·14492	761 ♯	Broadwood's 'Medium Pitch'
453·6	539·4	1·05355	903 ♯	Actual 9 C′, made by J. H. Griesbach for the Society of Arts
456·19	542·5	1·05957	1001 ♯	Mean of wind band of the Philharmonic, 1849–1874
458·4$\dot{6}$	545·2	1·06484	1088 ♯	Broadwood's 'High Pitch,' Philharmonic, July 1874

Table IV.

HARMONICS, AS INSERTED IN TABLE I, AFTER REDUCTION TO THE SAME OCTAVE, CALCULATED TO DIMES.

No.	Sem	No.	Sem	No.	Sem	No.	Sem
1	0	17	49·0496	33	60·5328	49	67.3765
2	12	18	50·0391	34	61·0496	50	67·7262
3	19·0196	19	50·9751	35	61·5514	51	68·0699
4	24	20	51·8631	36	62·0391	52	68·4053
5	27·8631	21	52·7079	37	62·5134	53	68·7351
6	31·0196	22	53·5132	38	62·9751	54	69·0587
7	33·6883	23	54·2827	39	63·4249	55	69·3763
8	36	24	55·0196	40	63·8631	56	69·6883
9	38·0391	25	55·7262	41	64·2906	57	69·9947
10	39·8631	26	56·4053	42	64·7074	58	70·2958
11	41·5132	27	57·0587	43	65·1152	59	70·5917
12	43·0196	28	57·6883	44	65·5132	60	70·8827
13	44·4053	29	58·2958	45	65·9022	61	71·1688
14	45·6883	30	58·8827	46	66·2827	62	71·4504
15	46·8827	31	59·4504	47	66·6551	63	71·7274
16	48	32	60	48	67·0196	64	72

See No. 5 of Directions appended to Table II. Reduce to the same Octave by subtracting 12, 24, 48, 60, or 72, so as to leave a remainder less than 12.

Table V.

THE 52 JUST NOTES IN THE DIAGRAM.

The small letters indicate Just Intonation. The table consists of four columns proceeding from bottom to top by just Fifths up (add 7·0196 sem, and to keep in the same Octave subtract 12 sem when necessary), and of 13 lines proceeding from left to right by just major Thirds up (add 3·8631 sem, and to keep in the same Octave subtract 12 where necessary). To complete the table for all the 117 tones of just intonation, three more columns on the left and two more on the right are required. (See my Appendix to Helmholtz, pp. 668–671.)

Against each note is written its pitch to a dime, the pitch of C being taken as 0 sem.

The notes with an inferior $_1$ and $_2$, as a_1, a_2 (read A-one, A-two), are respectively one and two commas (or 0·2151 and 0·4302 sem) flatter, and the notes with a superior ^{1}, as a^1 (read one-A) are one comma sharper, than the notes of the same name without numbers.

	Sem		Sem		Sem		Sem
a^1	9·2737	$c\sharp$	1·1369	$e_1\sharp$	5·0000	$g_2\sharp\sharp$	8·8631
d^1	2·2542	$f\sharp$	6·1173	$a_1\sharp$	9·9805	$c_2\sharp\sharp$	1·8436
g^1	7·2346	b	11·0978	$d_1\sharp$	2·9609	$f_2\sharp\sharp$	6·8240
c^1	0·2151	e	4·0782	$g_1\sharp$	7·9413	$b_2\sharp$	11·8048
f^1	5·1955	a	9·0587	$c_1\sharp$	0·9218	$e_2\sharp$	4·7849
$b^1\flat$	10·1760	d	2·0391	$f_1\sharp$	5·9022	$a_2\sharp$	9·7654
$e^1\flat$	3·1564	g	7·0196	b_1	10·8827	$d_2\sharp$	2·7458
$a^1\flat$	8·1369	C	0	e_1	3·8631	$g_2\sharp$	7·7263
$d^1\flat$	1·1173	f	4·9804	a_1	8·8436	$c_2\sharp$	0·7067
$g^1\flat$	6·0978	$b\flat$	9·9609	d_1	1·8240	$f_2\sharp$	5·6872
$c^1\flat$	11·0782	$e\flat$	2·9414	g_1	6·8045	b_2	10·6676
$f^1\flat$	4·0587	$a\flat$	7·9218	c_1	11·7849	e_2	3·6481
$b^1\flat\flat$	9·0391	$d\flat$	0·9022	f_1	4·7654	a_2	8·6285

Table VI.

TABLE OF THE NOTES ON DR. PREYER'S TONOMETER.

The notes are numbered 0, 1, to 32. Against each is placed the actual number of vibrations, and the pitch in relation to the lowest note 0, calculated to dimes. The numbers of the notes are given in the tables in the text.

No.	Vib	Sem	No.	Vib	Sem
0	127· 6	0			
1	131· 6	0·5342	17	195·59	7·3946
2	135·63	1·0568	18	199·59	7·7451
3	139·63	1·5586	19	203·59	8·0886
4	143·66	2·0525	20	207·53	8·4255
5	147·73	2·5361	21	211·65	8·7607
6	151·80	3·0065	22	215·51	9·0736
7	155·68	3·4435	23	219·51	9·3917
8	159·68	3·8828	24	223·48	9·7023
9	163·68	4·3108	25	227·48	10·0093
10	167·68	4·7289	26	231·41	10·3058
11	171·62	5·1312	27	235·33	10·5968
12	175·53	5·5210	28	239·33	10·8886
13	179·53	5·9113	29	243·35	11·1769
14	183·53	6·2928	30	247·35	11·4591
15	187·53	6·6659	31	251·23	11·7286
16	191·48	7·0207	32	255·30	12.0068

DISCUSSION.

Dr. STONE said there was so much of interest in Mr. Ellis's paper, that he could go on asking questions upon it for an hour. He was very glad that he and his friend Dr. Stainer had been so mistaken in judging of the two tuning-forks exhibited by Mr. Ellis, because he had always contended that absolute pitch could not be detected by the ear, and he was confirmed by Mr. Hipkins. One of the tuning-forks was much thinner in metal than the other and produced a thinner quality of tone, which to the ear sounded sharper. In the same way Mr. Hipkins told him he had two grand pianos absolutely in tune together, but one, which was somewhat softer in tone than the other, was always pronounced by musicians to be the flatter. There was a reason for this effect because the thinner fork being of thinner metal was more thrown into irregular vibrations on account of its shape, and produced a sharper effect on the hearer. (See P.S., p. 24.)

Mr. ELLIS said he had mentioned in the paper that it was desirable always to have exactly the same quality of tone, and as this was not the case with these two forks, it was not, of course, a fair trial.

Dr. STONE said he had gone into the question physiologically some time ago, and he found, certainly, that the absolute determination of pitch was impossible, but that it differed at different times and in different persons. Some persons even had different conceptions of unison. He knew a gentleman, a very good violoncello player, who always tuned a unison decidedly sharp. This was easily accounted for if the tympanum were a little thicker than ordinary, as was the case when one had a bad cold, for this condition affected the sensation of pitch very much. If then they were changing from day to day and from moment to moment, and some persons absolutely could not tell which was the sharper or flatter of two notes, that was a strong physiological argument against there being an absolute sense of pitch in the organs of sense.

Mr. ELLIS remarked that he had spoken particularly at the beginning of the paper of arithmetical, not physiological pitch.

Dr. STONE said he had always held that tuning-forks were very bad tests of pitch, because they were influenced so much by temperature and other causes. He wished to ask what was the test by which Mr. Ellis tried these forks?

Mr. ELLIS.—Mr. Appunn's tonometer.

Dr. STONE said the question was, *Quis custodiet ipsos custodes?*

Mr. ELLIS said he counted the beats and found the Octave was perfect, and then carefully tested several other intervals. The reed of the instrument was very slightly indeed affected by temperature, and it was mathematically impossible, if these intervals were correct, that the pitch could be other than that he assigned to it from counting the beats. All the intervals were proved in this way He did not trust his own ear. Mr. Hipkins and Mr. Greaves were both perfectly satisfied with the experiments.

Dr. Stone asked how the absolute pitch was determined from the beats.

Mr. Ellis said it was easily determined by a mathematical equation.

Dr. Stone said this was new to him, and was of very great importance. How was it that the French pitch was not determined in this way?

Mr. Ellis replied that at that time Koenig had not constructed his tonometer. Scheibler had made his, but it could not be obtained in France, and probably was not known. The discovery that absolute pitch could be determined mathematically from beats was due to Scheibler, but unfortunately it was not generally known at the time the French diapason was determined. That was quite wrong; but the Society of Arts pitch was worse. The discovery was published in 1834, but the forks could not be obtained. Koenig now made them, but Mr. Appunn's instrument was much easier to work with.

Dr. Stone said he was sure this discovery that absolute pitch could be determined from beats was very little known, and it was much to be desired that tuning-forks should be corrected by it as soon as possible. Another point he was going to mention was the physiological question with regard to membrane of Corti. He had never been able to see in reading Helmholtz's works, including his recent pamphlet, that he was justified in the assertion which he virtually made, that one fibre always vibrated to one note. If you tried on an ordinary instrument, striking one string between two others, they would both vibrate to a certain extent. With reference to the lowest audible pitch, it did not follow because the organ of Corti was not long enough to vibrate to a low note, that two of them vibrating consentaneously would not produce a different sensation, and one of a much lower note than any single one.

Dr. Stainer said he had once before stated that some musicians were gifted with a sense of absolute pitch, but he did not mean thereby, that they could state with precision how many vibrations per second any particular fork would give. What he meant was, that a great many people could tell within a very small interval, on hearing a note, whether it was *b*♭, *a*, *g*♯, or whatever the particular note was. He should like to know from Mr. Ellis if it was found that the ear improved in delicacy by listening to the same intervals several times in succession.

Mr. Ellis replied that Dr. Preyer stated that practice improved the ear very much indeed; it was particularly noticeable in attempts to assign the intervals of any high notes, where at first the guesses were perfectly wild.

Mr. W. H. Cummings said that neither he nor Dr. Verrinder had been deceived by the two forks which had misled Dr. Stone and Dr. Stainer. He concurred with Dr. Stone that the sense of pitch differed with the condition of the body. When out of sorts he generally heard out of tune, and of course it was almost impossible to sing in tune under such circumstances. He knew a case of a very clever musician, now a pensioner of the Royal Academy of Music, who was at one time a very admirable performer, but who had suffered from paralysis, and at the present moment he heard perfectly in tune with one ear, but the other ear was out to the extent of a semi-tone.

Mr. ELLIS remarked that this agreed to a considerable extent with the observations of Dr. Oscar Wolff, who had found a great difference in the different ears of the same perso —ith regard to their power of hearing and judging of intervals.

Mr. W. H. CUMMINGS said that some people imagined that it was impossible to distinguish exact pitch or intonation without education, but he had a case in his own family of a child, eight years old, who could tell any note struck upon a pianoforte in the next room.

The CHAIRMAN added that he had a sister who, at the age of six, could tell any note; so that it was a natural gift, not the result of education.

Dr. STONE remarked that even at six years old Mozart was a good player; so that if playing could be acquired, this faculty could be also.

Mr. W. H. CUMMINGS said the child he referred to had been taught.

Mr. STEPHENS said that he himself possessed this faculty, and so did his friend Dr. Stainer. It was well known that Dr. Crotch and several others had the same gift. When he was a child he could tell if his father put his arm on the pianoforte, what notes he struck and what he left out.

Dr. STAINER said he did not doubt the fact, only he believed it to be acquired, and not a natural gift.

Mr. ELLIS then exhibited four different C forks, which differed considerably in pitch; one being the copy sold by Cramer of the Society of Arts fork, which Mr. Sims Reeves stated he had sent to Hereford for the organ to be tuned to, and another, giving Broadwood's high pitch.

Dr. C. G. VERRINDER said he did not believe the organ was so tuned, and to show how easily persons might be deceived in this way, he narrated an anecdote of Sir George Smart. He had to superintend the building of an organ, and being very particular about the pitch, he took his own fork out of its case and gave it to the builder to tune the organ by. The builder put the fork on one side, and forgot all about it; the organ was voiced, tuned, and erected. When Sir George Smart came to ask for his fork, the voicer found he had made a mistake, but he soon with his file tuned the fork to the organ, and handed it back to Sir George Smart, who did not discover the trick until some time afterwards.